7th Grade Mathematics

2015-2016 Curriculum Map

Instruction and Course Alignment to the Common Core State Standards Content and Mathematical Practices

Course Unit Scope and Sequence

Unit Resources:

Planning and Pacing Guide
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Mathematics
Curriculum and Instruction
Tacoma Public Schools

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Curriculum Guide Unit Overview

	Unit	Description	Est Time
First Semester	1: Ratios and Proportional Relationships	Engage NY Grade 7 Module 1 Students build upon sixth grade reasoning of ratios and rates to formally define proportional relationships and the constant of proportionality. Students explore multiple representations of proportional relationships by looking at tables, graphs, equations, and verbal descriptions. Students extend their understanding about ratios and proportional relationships to compute unit rates for ratios and rates specified by rational numbers. The unit concludes with students applying proportional reasoning to identify scale factor and create a scale drawing.	25 - 30 days (complete at or around conferences)
	2. Rational Numbers	Engage NY Grade 7 Module 2 Students formed a conceptual understanding of integers through the use of the number line, absolute value, and opposites and extended their understanding to include the ordering and comparing of rational numbers. This unit uses the Integer Game: a card game that creates a conceptual understanding of integer operations and serves as a powerful mental model students can rely on during the module. Students build on their understanding of rational numbers to add, subtract, multiply, and divide signed numbers. Previous work in computing the sums, differences, products, and quotients of fractions serves as a significant foundation.	25 days (complete the first week of December)
	3. Expression and Equations	Engage NY Grade 7 Module 3 This unit consolidates and expands upon students' understanding of equivalent expressions as they apply the properties of operations to write expressions in both standard form and in factored form. They use linear equations to solve unknown angle problems and other problems presented within context to understand that solving algebraic equations is all about the numbers. Students use the number line to understand the properties of inequality and recognize when to preserve the inequality and when to reverse the inequality when solving problems leading to inequalities. They interpret solutions within the context of problems. Students extend their sixth-grade study of geometric figures and the relationships between them as they apply their work with expressions and equations to solve problems involving area of a circle and composite area in the plane, as well as volume and surface area of right prisms.	30 days (complete by end of Semester)

	Unit	Description	Est Time
Second Semester	4: Percent and proportional Relationships	Engage NY Grade 7 Module 4 Students deepen their understanding of ratios and proportional relationships from Unit 1 by solving a variety of percent problems. They convert between fractions, decimals, and percents to further develop a conceptual understanding of percent and use algebraic expressions and equations to solve multi-step percent problems. An initial focus on relating 100% to “the whole” serves as a foundation for students. Students begin the unit by solving problems without using a calculator to develop an understanding of the reasoning underlying the calculations. Material in early lessons is designed to reinforce students’ understanding by having them use mental math and basic computational skills. To develop a conceptual understanding, students use visual models and equations, building on their earlier work with these. As the lessons and topics progress and students solve multi-step percent problems algebraically with numbers that are not as compatible, teachers may let students use calculators so that their computational work does not become a distraction.	20 days (complete by the end of February)
	5: Statistics and Probability	Engage NY Grade 7 Module 5 Students begin their study of probability, learning how to interpret probabilities and how to compute probabilities in simple settings. They also learn how to estimate probabilities empirically. Probability provides a foundation for the inferential reasoning developed in the second half of this unit. Additionally, students build on their knowledge of data distributions that they studied in Grade 6, compare data distributions of two or more populations, and are introduced to the idea of drawing informal inferences based on data from random samples.	15 days (complete at or around conferences)
	6: Geometry	Engage NY Grade 7 Module 6, Engage NY Grade 8 Modules 2 & 3 This unit is a compilation of 7th and 8th grade Geometry standards. Students learn about translations, reflections, and rotations in the plane and, more importantly, how to use them to precisely define the concept of congruence. Throughout Topic A, on the definitions and properties of the basic rigid motions, students verify experimentally their basic properties and, when feasible, deepen their understanding of these properties using reasoning. All the lessons of Topic B demonstrate to students the ability to sequence various combinations of rigid motions while maintaining the basic properties of individual rigid motions. Students learn that congruence is just a sequence of basic rigid motions in Topic C, and Topic D begins the learning of Pythagorean Theorem. Students learn about dilation and similarity and apply that knowledge to a proof of the Pythagorean Theorem based on the Angle-Angle criterion for similar triangles. The module begins with the definition of dilation, properties of dilations, and compositions of dilations. One overarching goal of this module is to replace the common idea of “same shape, different sizes” with a definition of similarity that can be applied to geometric shapes that are not polygons, such as ellipses and circles.	50 days (complete by June 10)

Unit 1- Ratios and Proportional Relationships (ENY G7 Mod 1) 30 days				
Students build upon sixth grade reasoning of ratios and rates to formally define proportional relationships and the constant of proportionality. Students explore multiple representations of proportional relationships by looking at tables, graphs, equations, and verbal descriptions. Students extend their understanding about ratios and proportional relationships to compute unit rates for ratios and rates specified by rational numbers. The module concludes with students applying proportional reasoning to identify scale factor and create a scale drawing.				
Big Ideas	**Common Core State Standards**	**Standards for Mathematical Practice**	**Objectives**	**Materials/Resources Examples**
Topic A: **Proportional Relationships** In Lesson 1 of Topic A, students are reintroduced to the meanings of value of a ratio, equivalent ratios, rate, and unit rate through a collaborative work task where they record their rates choosing an appropriate unit of rate measurement. In Lesson 2, students conceptualize that two quantities are proportional to each other when there exists a constant such that each measure in the first quantity multiplied by this constant gives the corresponding measure in the second quantity (**7.RP.A.2**). They then apply this basic understanding in Lessons 3–6 by examining situations	**7.RP.A.2a** **A. Analyze proportional relationships and use them to solve real-world and mathematical problems.** **2.** Recognize and represent proportional relationships between quantities **a)** Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.	**MP.1 Make sense of problems and persevere in solving them.** Students make sense of and solve multistep ratio problems, including cases with pairs of rational number entries; they use representations, such as ratio tables, the coordinate plane, and equations, and relate these representations to each other and to the context of the problem. Students depict the meaning of constant proportionality in proportional relationships, the importance of (0,0) and (1,r) on graphs and the implications of how scale factors magnify or shrink actual lengths of figures on a scale drawing. **MP.2 Reason abstractly and quantitatively.** Students compute unit rates for paired data given in tables to determine if the data represents a proportional relationship. Use of concrete numbers will be analyzed to create	• Students compute unit rates associated with ratios of quantities measured in different units. Students use the context of the problem to recall the meaning of value of a ratio, equivalent ratios, rate, and unit rate, relating them to the context of the experience. • Students understand that two quantities are *proportional to* each other when there exists a constant (number) such that each measure in the first quantity multiplied by this constant gives the corresponding measure in the second quantity. • When students identify the measures in the first quantity with x and the measures in the second quantity with y, they will recognize that the second quantity is *proportional to* the first quantity if $y=kx$ for some positive number k. They apply this same relationship when using variable choices other than x and y. • Students examine situations to decide whether two quantities are proportional to each other by checking for a constant multiple between measures of x and measures of y when given in a table.	**ENY 7 Mod1** **Topic A** Lessons 1-6

to decide whether two quantities are in a proportional or non-proportional relationship by first checking for a constant multiple between measures of the two quantities, when given a table, and then by graphing on a coordinate plane. Students recognize that the graph of a proportional relationship must be a straight line through the origin (**7.RP.A.2a**).		and implement equations, including $y=kx$, where $k$is the constant of proportionality. Students decontextualize a given constant speed situation, representing symbolically the quantities involved with the formula, $distance=rate\times time$. In scale drawings, scale factors will be changed to create additional scale drawings of a given picture.	• Students study examples of relationships that are not proportional in addition to those that are. • Students decide whether two quantities are proportional to each other by graphing on a coordinate plane and observing whether the graph is a straight line through the origin. • Students study examples of quantities that are proportional to each other as well as those that are not. • Students examine situations carefully to decide whether two quantities are proportional to each other by graphing on a coordinate plane and observing whether all the points would fall on a line that passes through the origin. • Students study examples of relationships that are not proportional as well as those that are.	
Topic B: **Unit Rate and constant of Proportionality** In Topic B, students learn to identify the constant of proportionality by finding the unit rate in the collection of equivalent ratios. They represent this relationship with equations of the form	**7.RP.A.2b, 2c, 2d** **A. Analyze proportional relationships and use them to solve real-world and mathematical problems.** **2**. Recognize and represent proportional relationships between quantities. **b)** Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. **c)** Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant*		• Students identify the same value relating the measures of x and the measures of y in a proportional relationship as the constant of proportionality and recognize it as the unit rate in the context of a given situation. • Students find and interpret the constant of proportionality within the contexts of problems. • Students use the constant of proportionality to represent proportional relationships by equations in real- world contexts as they relate the equations to a	**ENY 7 Mod1** **Topic B** Lessons 7-10 The Mid-Module Assessment follows Topic B. Mid Module Assessment

$y=kx$, where k is the constant of proportionality (**7.RP.A.2**, **7.RP.A.2c**). In Lessons 8 and 9, students derive the constant of proportionality from the description of a real-world context and relate the equation representing the relationship to a corresponding ratio table or graphical representation (**7.RP.A.2b**, **7.EE.B.4**). Topic B concludes with students consolidating their graphical understandings of proportional relationships as they interpret the meanings of the points (0,0) and (1,r), where r is the unit rate, in terms of the situation or context of a given problem (**7.RP.A.2d**).	*price p, the relationship between the total cost and the number of items can be expressed as t = pn*. **d)** Explain what a point (*x*, *y*) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r*) where r is the unit rate. **7.EE.B.4a** Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. **a)** Solve word problems leading to equations of the form *px* + *q* = *r* and *p*(*x*+ *q*) = *r*, where *p*, *q*, and *r* are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*		corresponding ratio table and/or graphical representation. • Students consolidate their understanding of equations representing proportional relationships as they interpret what points on the graph of a proportional relationship mean in terms of the situation or context of the problem, including the point (0,0). • Students are able to identify and interpret in context the point (1,) on the graph of a proportional relationship where r is the unit rate.	
Topic C: **Ratios and Rates involving Fractions** In the first two lessons of Topic C, students' knowledge of unit rates for ratios and rates is extended	**7.RP.A.1** **A.** Analyze proportional relationships and use them to solve real-world and mathematical problems. **1.** Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks*		• Students use ratio tables and ratio reasoning to compute unit rates associated with ratios of fractions in the context of measured quantities such as recipes, lengths, areas, and speed. • Students work collaboratively to solve a problem while sharing their	**ENY 7 Mod 1** **Topic C** Lessons 11-15 12 is Optional Omit example 1 lesson 13

by considering applications involving fractions, such as a speed of 12 mile per 14 hour. Students continue to use the structure of ratio tables to reason through and validate their computations of rate. In Lesson 13, students continue to work with ratios involving fractions as they solve problems where a ratio of two parts is given along with a desired total quantity. Students can choose a representation that most suits the problem and their comfort levels, such as tape diagrams, ratio tables, or possibly equations and graphs, as they solve these problems, reinforcing their work with rational numbers. In Lesson 14, students solve multi-step ratio problems, which include fractional markdowns, markups, commissions, and fees. In the final lesson of the topic, students focus their attention on using equations and graphs	*1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction* $^{1/2}/_{1/4}$ *miles per hour, equivalently 2 miles per hour*. **7.RP.A.3** **3.** Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error. **7.EE.B.4a** **4.** Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. **a)** Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*		thinking processes, strategies, and solutions with the class. • Students use tables to find an equivalent ratio of two partial quantities given a part-to-part ratio and the total of those quantities, in the third column, including problems with ratios of fractions. • Students solve multi-step ratio problems including fractional markdowns, markups, commissions, fees, etc. • Students use equations and graphs to represent proportional relationships arising from ratios and rates involving fractions. • Students interpret what points on the graph of the relationship mean in terms of the situation or context of the problem.	

to represent proportional relationships involving fractions, reinforcing the process of interpreting the meaning of points on a graph in terms of the situation or context of the problem.				
Topic D: Ratios of Scale Drawings In the first lesson of Topic D, students are introduced to scale drawings; they determine if the drawing is a reduction or enlargement of a two-dimensional picture. Pairs of figures are presented for students to match corresponding points. In Lesson 17, students learn the term *scale factor* and recognize it as the constant of proportionality. With a given scale factor, students make scale drawings of pictures or diagrams. In Lessons 18 and 19, students compute the actual dimensions of objects shown in	**7.RP.A.2b** **Analyze proportional relationships and use them to solve real-world and mathematical problems.** **2.** Recognize and represent proportional relationships between quantities **b)** Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. **7.G.A.1** **A. Draw construct, and describe geometrical figures and describe the relationships between them.** **1**. Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.		• Students understand that a scale drawing is either the reduction or the enlargement of a two-dimensional picture. • Students compare the scale drawing picture with the original picture and determine if the scale drawing is a reduction or an enlargement. • Students match points and figures in one picture with points and figures in the other picture. • Students recognize that the enlarged or reduced distances in a scale drawing are proportional to the corresponding distances in the original picture. • Students recognize the scale factor to be the constant of proportionality. • Given a picture or description of geometric figures, students make a scale drawing with a given scale factor. • Given a scale drawing, students compute the lengths in the actual picture using the scale. Students identify the scale factor in order to make intuitive comparisons of size,	**ENY Mod 1** **Topic D** Lessons 16-18 Lesson 20-22 (if time or high achieving differentiation) The End-of-Module Assessment follows Topic D. End of Module Assessment

pictures given the scale factor. They recognize that areas scale by the square of the scale factor that relates lengths. In the final lessons, students engage in their own scale factor projects—first, to produce a scale drawing of the top-view of a furnished room or building, and second, given one scale drawing, to produce new scale drawing using a different scale factor.			and then devise a strategy for efficiently finding actual lengths using the scale.	

Unit 2 – Rational Numbers and Exponents (ENY G7 Mod 2) Time Line 25 days				
Students have formed a conceptual understanding of integers through the use of the number line, absolute value, and opposites and extended their understanding to include the ordering and comparing of rational numbers. This unit uses the Integer Game: a card game that creates a conceptual understanding of integer operations and serves as a powerful mental model students can rely on during the module. Students build on their understanding of rational numbers to add, subtract, multiply, and divide signed numbers. Previous work in computing the sums, differences, products, and quotients of fractions serves as a significant foundation.				
Big Ideas	**Common Core State Standards**	**Standards for Mathematical Practice**	**Objectives**	**Materials/Resources Examples**
Topic A: **Addition and Subtraction of Integers and Rational Numbers** In Topic A, students find sums and differences of signed numbers and establish rules related to the addition and subtraction of rational numbers (**7.NS.A.1**). Students draw upon experiences in modeling, ordering, and comparing integers and other rational numbers from Grade 6, Module 3 (**6.NS.C.5**, **6.NS.C.6**, **6.NS.C.7**). They use their previous work with adding and subtracting fractions and decimals (**5.NF.A.1**, **6.NS.B.3**) to compute the sums and differences of rational numbers. In Lesson 1, students play a card game called the Integer Game to understand how a number and its opposite combine to make zero. The number line is used to count up and down, serving as a visual model for finding sums. In Lessons 2 and 3, students more formally develop their understanding of the addition of integers. They use vectors to represent integers on the number line and apply the concept of absolute value (**6.NS.C.7c**) to represent the length of the vector while interpreting the sign of the integer as the vector's direction. By	**7.NS.A.1** **A. Apply and extend previous understandings of operations with fractions.** **1.** Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. **a)** Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged*. **b)** Understand $p + q$ as the number located a distance $\|q\|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.\\ **c)** Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$.	MP.1 **Make sense of problems and persevere in solving them.** When problem-solving, students use a variety of techniques to make sense of a situation involving rational numbers. For example, they may draw a number line and use arrows to model and make sense of an integer addition or subtraction problem. Or when converting between forms of rational numbers, students persevere in carrying out the long division algorithm to determine a decimal's repeat pattern. A tape diagram may be constructed as an entry point to make sense of a working-backwards problem. As students fluently solve word problems using algebraic equations and	• Students add positive integers by counting up and negative integers by counting down (using curved arrows on the number line). • Students play the Integer Game to combine integers, justifying that an integer plus its opposite add to zero. • Students know the opposite of a number is called the additive inverse because the sum of the two numbers is zero. • Students use properties of operations to add and subtract rational numbers without the use of a calculator. • Students recognize that any problem involving addition and subtraction of rational numbers can be written as a problem using addition and subtraction of positive numbers only. • Students use the commutative and associative properties of	**ENY 7 Mod 2** **Topic A** Integer Game & Lessons 1-9 Integer Game (one day) • Find the integer game before lesson 1-found in teacher book only Lessons 1 and 8 • Lesson 4 optional • After lesson 8 (teacher page 97) Included- in the student work book the lesson sprint. (adding of integers) Lesson 9 • After lesson 9 (teacher page 109) Included- in the student work book the lesson sprint. (subtraction of integers)

Lesson 4, students are efficiently adding integers using well-defined rules. After addition rules are formalized, students begin subtracting integers in Lesson 5. They relate subtraction to removing a card from their hand in the Integer Game, realizing that subtracting a positive card has the same effect as adding or picking up a negative card. Similarly, removing (subtracting) a negative card increases students' scores the same way as adding the corresponding positive card. Therefore, students determine that subtracting a signed number is the same as adding its opposite. In Lesson 6, students deepen their understanding of subtraction using absolute value and the number line to justify that the distance between two signed numbers is the absolute value of their difference. They represent sums and differences of rational numbers using the number line in Lesson 7 and use vectors to model the sum, $p+q$, or the difference, $p-q$. As Topic A concludes, students apply the properties of operations to add and subtract rational numbers in Lessons 8 and 9. Using the properties of operations and their fluency in adding and subtracting decimals and fractions from earlier grades, they rewrite numerical expressions in different forms to efficiently find sums and differences of signed numbers without the use of a calculator.	Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. **d)** Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. **e)** Apply properties of operations as strategies to add and subtract rational numbers.	inverse operations, they consider their steps and determine whether or not they make sense in relationship to the arithmetic reasoning that served as their foundation in earlier grades. MP.2 **Reason abstractly and quantitatively**. Students make sense of integer addition and subtraction through the use of an integer card game and diagramming the distances and directions on the number line. They use different properties of operations to add, subtract, multiply, and divide rational numbers, applying the properties to generate equivalent expressions or explain a rule. Students use integer subtraction and absolute value to justify the distance between two numbers on the number line. Algebraic expressions and equations are created to represent relationships. Students know how to use the	addition to rewrite numerical expressions in different forms. They know that the opposite of a sum is the sum of the opposites (e.g., $-3+(-4)=-3+4$).	

Topic B: Multiplication and Division of Integers and Rational Numbers In Topic B, students extend their understanding of multiplication and division of whole numbers, decimals, and fractions to find the products and quotients of signed numbers (**7.NS.A.2**). Students begin in Lesson 10 by returning to conceptualization of multiplication as repeated addition. They relate multiplication to the Integer Game. For instance, gaining four −5 cards, or 4(−5), is the same as 0+(−5)+(−5)+(−5)+(−5), which is the same as 0−5−5−5−5, or −20. They realize that if a negative card is taken out of their hand multiple times, their score goes up, for example, (−2)(−6)=0−(−6)−(−6)=0+6+6=12. In Lesson 11, students draw upon their experiences with the integer card game to justify the rules for multiplication of integers. The additive inverse (**7.NS.A.1c**) and distributive property are used to show that (−1)(−1)=1 (**7.NS.A.2a**). From earlier grades, students understand division as the process of finding the missing factor of a product (**3.OA.B.6**). In Lesson 12, they use this relationship to justify that the rules for dividing signed numbers are consistent with that of multiplication, provided the divisor is not zero (**7.NS.A.2b**). Students extend the integer rules to include all rational numbers, recognizing that every quotient of two integers is a rational number provided the divisor is not zero. In Lesson 13, students realize that the context of a word problem often determines whether the answer should be expressed in the fractional or	**7.NS.A.2** **A.** Apply and extend previous understanding of operations with fractions to add, subtract, multiply, and divide rational numbers. **2**. Apply and extend previous understanding of multiplication and division and of fractions to multiply and divide rational numbers. ***a)** Understand that multiplication is extended from fractions to rational numbers by requiring that operation continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (-1)(-1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. **b**) Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then -(p/q) = (-p)/q = p/(-q). Interpret quotients of rational numbers by describing real-world contexts. **c**) Apply properties of operations as strategies to multiply and divide rational numbers. **d**) Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.	properties of operations to solve equations. They make “zeros and ones” when solving an algebraic equation, thereby demonstrating an understanding of how the use of inverse operations ultimately leads to the value of the variable. **MP.4 Model with mathematics**. Through the use of number lines, tape diagrams, expressions, and equations, students model relationships between rational numbers. Students relate operations involving integers to contextual examples. For instance, an overdraft fee of $25 that is applied to an account balance of −$73.06, is represented by the expression −73.06−25 or −73.06+(−25) using the additive inverse. Students compare their answers and thought processes in the Integer Game and use number line diagrams to ensure accurate reasoning. They deconstruct a difficult word problem by writing an equation, drawing a number line, or drawing a tape diagram to	• Students practice and justify their understanding of multiplication of integers by using the Integer Game. For example, 3×5 corresponds to what happens to your score if you get three 5 cards; 3×(−5) corresponds to what happens to your score if you get three −5 cards; (−3)×5 corresponds to what happens to your score if you lose three 5 cards; and (−3)×(−5) corresponds to what happens to your score if you lose three −5 cards. • Students explain that multiplying by a positive integer is repeated addition and that adding a number multiple times has the same effect as removing the opposite value the same number of times (e.g., 5×3=(−5)×(−3) and 5×(−3)=(−5)×3). • Students use the properties and facts of operations to extend multiplication of whole numbers to multiplication of integers. • Students understand the rules for multiplication of integers and that multiplying the absolute values of integers results in the absolute value of the product. The sign, or absolute value, of the	**ENY 7 Mod 2** **Topic B:** Lessons 10-12 • After lesson 12 (teacher page 139) Included- in the student work book the lesson sprint (dividing integers) Lessons 13-16 • After lesson 16 (teacher page 178) included- in the student work book the lesson sprint (dividing integers) Notes: • **Lesson 13:** students need confidence with exponents, if student skills are not here, omit this strategy but teacher conversions. • **Lesson 16:** the purposes of this lesson is to introduce the distributive property, use your discretion. After Topic B Mid Module Assessment Mid Module Assessment

decimal form of a rational number. They draw upon their previous understanding of equivalent fractions, place value, and powers of ten to convert fractions whose denominators are a product of 2s and 5s into decimals. In Lesson 14, students use long division to convert any fraction into a decimal that either terminates in zeros or repeats (**7.NS.A.2d**). Products and quotients continue to be related to the real world. In Lesson 15, students create numerical expressions with rational numbers based on the context of word problems. In Lesson 16, properties of operations are used to rewrite expressions in equivalent forms as students multiply and divide rational numbers efficiently without the aid of a calculator (**7.NS.A.2c**).		represent quantities. To find a change in elevation, students may draw a picture representing the objects and label their heights to aid in their understanding of the mathematical operation(s) that must be performed. **MP.6 Attend to precision.** In performing operations with rational numbers, students understand that the decimal representation reflects the specific place value of each digit. When converting fractions to decimals, they carry out their calculations to specific place values, indicating a terminating or repeating pattern. In stating answers to problems involving signed numbers, students use integer rules and properties of operations to verify that the sign of their answer is correct. For instance, when finding an average temperature for temperatures whose sum is a negative number, students realize that the quotient must be a negative number since the divisor is positive and	product is positive if the factors have the same sign and negative if they have opposite signs. • Students realize that $(-1)(-1)=(1)$ and see that it can be proven mathematically using the distributive property and the additive inverse. • Students use the rules for multiplication of signed numbers and give real-world examples. • Students recognize that division is the reverse process of multiplication, and that integers can be divided provided the divisor is not zero. • Students understand that every quotient of integers (with a non-zero divisor) is a rational number and divide signed numbers by dividing their absolute values to get the absolute value of the quotient. The quotient is positive if the divisor and dividend have the same signs and negative if they have opposite signs. • Students understand that the context of a real-life situation often determines whether a rational number should be represented as a fraction or decimal. • Students understand that decimals specify points on the number line by repeatedly subdividing	

		the dividend is negative. **MP.7 Look for and make use of structure.** Students formulate rules for operations with signed numbers by observing patterns. For instance, they notice that adding −7 to a number is the same as subtracting 7 from the number, and thus, they develop a rule for subtraction that relates to adding the inverse of the subtrahend. Students use the concept of absolute value and subtraction to represent the distance between two rational numbers on a number line. They use patterns related to the properties of operations to justify the rules for multiplying and dividing signed numbers. The order of operations provides the structure by which students evaluate and generate equivalent expressions.	intervals into tenths (*deci-* means one-tenth). • Students convert positive decimals to fractions and fractions to decimals when the denominator is a product of only factors of either 2 or 5. • Students understand that every rational number can be converted to a decimal. • Students represent fractions as decimal numbers that either terminate in zeros or repeat. Students then also represent repeating decimals using a bar over the shortest sequence of repeating digits. • Students interpret word problems and convert between fraction and decimal forms of rational numbers. • Students recognize that the rules for multiplying and dividing integers apply to rational numbers. • Students interpret products and quotients of rational numbers by describing real-world contexts. • Students use properties of operations to multiply and divide rational numbers without the use of a calculator. They use the commutative and associative properties of multiplication to generate	

			equivalent expressions. They use the distributive property of multiplication over addition to create equivalent expressions, representing the sum of two quantities with a common factor as a product, and vice-versa. • Students recognize that any problem involving multiplication and division can be written as a problem involving only multiplication. • Students determine the sign of an expression that contains products and quotients by checking whether the number of negative terms is even or odd.	

Topic C: Applying Operations with Rational Numbers to Expressions and Equations Students use algebra and rational numbers in Topic C to problem-solve, building upon their foundational work with rational numbers and expressions and equations in Grade 6 (**6.NS.C.5**, **6.EE.A.2**, **6.EE.A.3**, **6.EE.A.4**, **6.EE.B.5**, **6.EE.B.6**, **6.EE.B.7**). Topic C begins in Lesson 17 with students finding solutions to word problems by working backwards and using tape diagrams to model the algebraic steps they use to arrive at the solution. In Lessons 18 and 19, students create and evaluate equivalent forms of expressions involving rational numbers to see structure, reveal characteristics, and make connections to context (**7.EE.A.2**). Lesson 20 is a modeling lesson in which students are presented with a scenario related to an investment account's activity over the course of several years. Students interpret the information and develop a strategy to find the actual changes to the account balance each year. In Lesson 21, students return to the Integer Game that they played in earlier lessons to better understand *if-then* statements. They relate making the same changes to two equal card-hand totals to making equivalent changes to each side of a true number sentence. Therefore, they show, for instance: *If* $a=b$, *then* $a-c=b-c$. Topic C concludes with a two-day lesson. In Lessons 22 and 23, students work towards fluently solving word problems through the use of equations (**7.EE.B.4a**). Using algebra to deconstruct and solve contextual	**7.NS.A.3** **A.** Apply and extend previous understandings of operations with fractions. **3.** Solve real-world and mathematical problems involving the four operations with rational numbers[1]. [1] *Computations with rational numbers extend the rules for manipulating fractions to complex fractions.* **7.EE.A.2** A. Use properties of operations to generate equivalent expressions. ***2.*** Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05".* **7.EE.B.4a** **B**. Solve real-life and mathematical problems using numerical and algebraic expressions and equations. **4**. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about quantities. **a)** Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where *p, q,* and *r* are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an		• Students use tape diagrams to solve equations of the form px + q = r and $(x+q)=r$, (where p, q, and r are *small positive* integers), and identify the sequence of operations used to find the solution. • Students translate word problems to write and solve algebraic equations using tape diagrams to model the steps they record algebraically. • Students create equivalent forms of expressions in order to see structure, reveal characteristics and make connections to context. • Students compare equivalent forms of expressions and recognize that there are multiple ways to represent the context of a word problem. • Students write and evaluate expressions to represent real-world scenarios. • Students use algebra to solve equations (of the form px$+q=r$ and $(x+q)=r$, where p, q, and r are specific rational numbers); using techniques of making zero (adding the additive inverse) and making one (multiplying by the multiplicative	**ENY 7 Mod 2** **Topic C:** Lessons 17-19 Lessons 22-23 End of Topic C End of Module Assessment End of Module Assessment

problems continues as the focus in Module 3.	arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its* **length** *is 6 cm. What is its width?*		inverse) to solve for the variable. • Students identify and compare the sequence of operations used to find the solution to an equation algebraically, with the sequence of operations used to solve the equation with tape diagrams. They recognize the steps as being the same. • Students solve equations for the value of the variable using inverse operations; by making zero (adding the additive inverse) and making one (multiplying by the multiplicative inverse).	

Unit 3 – Expression and Equations (ENY G7 Mod 3) Time Line 30 days (complete by end of January)				
This unit consolidates and expands upon students' understanding of equivalent expressions as they apply the properties of operations to write expressions in both standard form and in factored form. They use linear equations to solve unknown angle problems and other problems presented within context to understand that solving algebraic equations is all about the numbers. Students use the number line to understand the properties of inequality and recognize when to preserve the inequality and when to reverse the inequality when solving problems leading to inequalities. They interpret solutions within the context of problems. Students extend their sixth-grade study of geometric figures and the relationships between them as they apply their work with expressions and equations to solve problems involving area of a circle and composite area in the plane, as well as volume and surface area of right prisms.				
Big Ideas	**Common Core State Standards**	**Standards for Mathematical Practice**	**Objectives**	**Materials/Resources Examples**
Topic A: Use Properties of Operations to Generate Equivalent Expressions In Lesson 1 of Topic A, students write equivalent expressions by finding sums and differences extending the *any order* (commutative property) and *any grouping* (associative property) to collect like terms and rewrite algebraic expressions in standard form (**7.EE.A.1**). In Lesson 2, students rewrite products in standard form by applying the commutative property to rearrange like items (numeric coefficients, like variables) next to each other and rewrite division as multiplying by the multiplicative inverse. Lessons 3 and 4 have students using a rectangular array and the distributive property as they first multiply one term by a sum of two or more terms to expand a product to a sum, and then reverse the process to rewrite the sum as a product of the GCF and a remaining factor. Students model real-world problems with expressions and see how writing in one form versus another helps them to understand how the quantities are	**7.EE.A.1, 2** **A.** Use properties of operations to generate equivalent expressions. **1.** Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. **2.** Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example,* $aa+0.05aa=1.05aa$ *means that "increase by* 5%*" is the same as "multiply by* 1.05*."*	**MP.2 Reason abstractly and quantitatively.** Students make sense of how quantities are related within a given context and formulate algebraic equations to represent this relationship. They use the properties of operations to manipulate the symbols that are used in place of numbers, in particular, pi. In doing so, students reflect upon each step in solving and recognize that these properties hold true since the variable is really just holding the place for a number. Students analyze solutions and connect back to ensure reasonableness within context.	• Students generate equivalent expressions using the fact that addition and multiplication can be done in *any order* (commutative property) and *any grouping* (associative property). • Students recognize how *any order, any grouping* can be applied in a subtraction problem by using additive inverse relationships (adding the opposite) to form a sum and likewise with division problems by using the multiplicative inverse relationships (multiplying by the reciprocal) to form a product. • Students recognize that *any order* does not apply to expressions mixing addition and multiplication, leading to the need to follow the order of operations.	**ENY 7 Mod 3** **Topic A** Lessons 1-6 Include sprint teacher page 44

related (**7.EE.A.2**). In Lesson 5, students recognize that detecting inverses and the identity properties of 0 for addition and 1 for multiplication allows for ease in rewriting equivalent expressions. This topic culminates with Lesson 6 with students applying repeated use of the distributive property as they collect like terms containing fractional coefficients to rewrite rational number expressions.		**MP.4 Model with mathematics**. Throughout the module, students use equations and inequalities as models to solve mathematical and real-world problems. In discovering the relationship between circumference and diameter in a circle, they will use real objects to analyze the relationship and draw conclusions. Students test conclusions with a variety of objects to see if the results hold true, possibly improving the model if it has not served its purpose. **MP.6 Attend to precision**. Students are precise in defining variables. They understand that a variable represents one number. They use appropriate vocabulary and terminology when communicating about expressions, equations, and inequalities. They use the definition of equation from Grade 6 to understand how to use the equal sign consistently and appropriately. Circles and related notions about circles are	• Students use an area model to write products as sums and sums as products. • Students use the fact that the opposite of a number is the same as multiplying by −1 to write the opposite of a sum in standard form. • Students recognize that rewriting an expression in a different form can shed light on the problem and how the quantities in it are related. • Students recognize the identity properties of 0 and 1 and the existence of inverses (opposites and reciprocals) to write equivalent expressions. • Students rewrite rational number expressions by collecting like terms and combining them by repeated use of the distributive property.	

		precisely defined in this module. **MP.7 Look for and make use of structure.** Students recognize the repeated use of the distributive property as they write equivalent expressions. Students recognize how equations leading to the form $px+q=r$ and $(x+q)=r$ are useful in solving a variety of problems. They see patterns in the way that these equations are solved. Students apply this structure as they understand the similarities and differences in how an inequality of the type $px+q>r$ or $px+q<r$ is solved. **MP.8 Look for and express regularity in repeated reasoning**. Students use area models to write products as sums and sums as products and recognize how this model is a way to organize results from repeated use of the distributive property. As students work to solve problems, they maintain oversight of the process, while attending to the details. They continually		

		evaluate the reasonableness of solutions as they are represented in contexts that allow for students to know that they found the intended value for a given variable. As they solve problems involving pi, they notice how a problem may be reduced by using a given estimate for pi to make calculations more efficient.		
Topic B: Solve Problems Using Expressions, Equations, and Inequalities Topic B begins in Lesson 7 with students evaluating equations and problems modeled with equations for given rational number values to determine whether the value makes a true or false number sentence. In Lessons 8 and 9, students are given problems of perimeter; total cost; age comparisons; and distance, rate, and time to solve. Students will discover that modeling these types of problems with an equation becomes an efficient approach to solving the problem, especially when the problem contains rational numbers (**7.EE.B.3**, **7.EE.B.4a**). Students apply the properties of equality to isolate the variable in these equations as well as those created to model missing angle problems in Lessons 10 and 11. All problems provide a real-world or mathematical context so that students can connect the (abstract) variable, or letter, to the number that it actually represents in the problem. The number	**7.EE.B.3 & 4** **B. Solve real-life and mathematical problems using numerical and algebraic expressions and equations**. **3.** Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will*		• Students understand that an equation is a statement of equality between two expressions. • Students build an algebraic expression using the context of a word problem and use that expression to write an equation that can be used to solve the word problem. • Students understand and use the addition, subtraction, multiplication, division, and substitution properties of equality to solve word problems leading to equations of the form $px+q=r$ and $p(x+q)=r$, where p, q, and r are specific rational numbers. • Students understand that any equation can be rewritten as an equivalent equation with expressions that involve only integer	**Topic B** **Lesson 7-15** Omit lesson 8 Lesson 9 is a group game activity all materials in teacher book. Omit lesson 10 and 11, concepts will be covered during geometry units (7 Mod 6) After lesson 12 (teacher page 179) Included- in the student work book the lesson sprint. (equations) After lesson 15 (teacher page 213) Included- in the student work book the lesson sprint. (Inequalities) After Topic B Mid Module Assessment

already exists; students just need to find it. Lesson 12 introduces students to situations that are modeled in the form $px+q>r$ and $px+q<r$. Initially, students start by translating from verbal to algebraic, choosing the inequality symbol that best represents the given situation. Students then find the number(s) that make each inequality true. To better understand how to solve an inequality containing a variable, students look at statements comparing numbers in Lesson 13. They discover when (and why) multiplying by a negative number reverses the inequality symbol when this symbol is preserved. In Lesson 14, students extend the idea of isolating the variable in an equation to solve problems modeled with inequalities using the properties of inequality. This topic concludes with students modeling inequality solutions on a number line and interpreting what each solution means within the context of the problem (**7.EE.B.4b**).	*need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation*. 4. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. **7.G.B.5** **B. Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.** **5.** Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.		coefficients by multiplying both sides by the correct number. • Students justify the properties of inequalities that are denoted by < (less than), ≤ (less than or equal), > (greater than), and ≥ (greater than or equal). • Students understand that an inequality is a statement that one expression is less than (or equal to) or greater than (or equal to) another expression, such as $2x+3<5$ or $3x+50\geq100$. • Students interpret a solution to an inequality as a number that makes the inequality true when substituted for the variable. • Students convert arithmetic inequalities into a new inequality with variables (e.g., $2\times6+3>12$ to $2m+3>12$) and give a solution, such as $m=6$, to the new inequality. They check to see if different values of the variable make an inequality true or false. • Students graph solutions to inequalities taking care to interpret the solutions in the context of the problem.	Mid Module Assessment

Topic C: Use Equations and Inequalities to Solve Geometry Problems Topic C begins with students discovering the greatest ratio of all, pi. In Lesson 16, students use a compass to construct a circle and extend their understanding of angles and arcs from earlier grades to develop the definition of a circle through exploration. A whole-group activity follows, in which a wheel, chalk, and string are used to physically model the ratio of a circle's circumference to its diameter. Through this activity, students conceptualize pi as a number whose value is a little more than 3. The lesson continues to examine this relationship between a circle's circumference and diameter, as students understand pi to be a constant, which can be represented using approximations. Students see the usefulness of approximations such as 227 and 3.14 to efficiently solve problems related to the circumference of circles and semicircles. Students continue examining circles in Lesson 17, as they discover what happens if they cut a circle of radius length r into equivalent-sized sectors and rearrange them to resemble a rectangle. Applying what they know about the area of a rectangle, students examine the dimensions to derive a formula for the area of the circle (**7.G.B.4**). They use this formula, A=πr2, to solve problems with circles. In Lesson 18, students consider how to adapt the area and circumference formulas to examine interesting problems involving *quarter circle* and *semicircle* regions. Students	**7.G.B.4 & 6** **B.** Solve real-life and mathematical problems involving angle measure, area, surface area, and volume. **4,** Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle. **6.** Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.		• Students develop the definition of a circle using diameter and radius. • Students know that the distance around a circle is called the *circumference* and discover that the ratio of the circumference to the diameter of a circle is a special number called pi, written π. • Students know the formula for the circumference C of a circle of diameter d and radius r. They use scale models to derive these formulas. • Students use 3.14 as estimates for π and informally show that π is slightly greater than 3. • Students give an informal derivation of the relationship between the circumference and area of a circle. • Students know the formula for the area of a circle and use it to solve problems. • Students examine the meaning of *quarter circle* and *semicircle*. • Students solve area and perimeter problems for regions made out of rectangles, quarter circles, semicircles, and circles, including solving for unknown lengths when	**Mod 3 Topic C** Lessons 16-26 Purpose is equation and formula development NOT MASTERY-Use at teacher discretion to depth learning. After Topic C End of Module Assessment End of Module Assessment

analyze figures to determine composite area in Lesson 19 and 20 by composing and decomposing into familiar shapes. They use the coordinate plane as a tool to determine the length and area of figures with vertices at grid points. This topic concludes as students apply their knowledge of plane figures to find the surface area and volume of three-dimensional figures. In Lessons 21 and 22, students will use polyhedron nets to understand surface area as the sum of the area of the lateral faces and the area of the base(s) for figures composed of triangles and quadrilaterals. In Lessons 23 and 24, students will recognize the volume of a right prism to be the area of the base times the height and compute volumes of right prisms involving fractional values for length (**7.G.B.6**). In the last two lessons, students solidify their understanding of two- and three-dimensional objects as they solve real-world and mathematical problems involving area, volume, and surface area.			the area or perimeter is given. • Students find the areas of triangles and simple polygonal regions in the coordinate plane with vertices at grid points by composing into rectangles and decomposing into triangles and quadrilaterals. • Students find the area of regions in the coordinate plane with polygonal boundaries by decomposing the plane into triangles and quadrilaterals, including regions with polygonal holes. • Students find composite area of regions in the coordinate plane by decomposing the plane into familiar figures (triangles, quadrilaterals, circles, semi-circles, and quarter circles). • Students find the surface area of three-dimensional objects whose surface area is composed of triangles and quadrilaterals. They use polyhedron nets to understand that surface area is simply the sum of the area of the lateral faces and the area of the base(s). • Students use the known formula for the volume of	

			a right rectangular prism (length×width×height). • Students understand the volume of a right prism to be the area of the base times the height. • Students compute volumes of right prisms involving fractional values for length. • Students use the formula for the volume of a right rectangular prism to answer questions about the capacity of tanks. • Students solve real-world and mathematical problems involving volume and surface areas of three-dimensional objects composed of cubes and right prisms.	

Unit 4 – Percent and Proportional Relationships (ENY G7 Mod 4) Time Line - 20 days				
Students deepen their understanding of ratios and proportional relationships from Module 1 by solving a variety of percent problems. They convert between fractions, decimals, and percents to further develop a conceptual understanding of percent and use algebraic expressions and equations to solve multi-step percent problems. An initial focus on relating 100% to "the whole" serves as a foundation for students. Students begin the module by solving problems without using a calculator to develop an understanding of the reasoning underlying the calculations. Material in early lessons is designed to reinforce students' understanding by having them use mental math and basic computational skills. To develop a conceptual understanding, students use visual models and equations, building on their earlier work with these. As the lessons and topics progress and students solve multi-step percent problems algebraically with numbers that are not as compatible, teachers may let students use calculators so that their computational work does not become a distraction.				
Big Ideas	**Common Core State Standards**	**Standards for Mathematical Practice**	**Objectives**	**Materials/Resources Examples**
Topic A: Finding the Whole Topic A builds on students' conceptual understanding of percent from Grade 6 (**6.RP.A.3c**) and relates 100% to "the whole." Students represent percents as decimals and fractions and extend their understanding from Grade 6 to include percents greater than 100%, such as 225%, and percents less than 1%, such as $\frac{1}{2}$ % or 0.5%. They understand that, for instance, 225% means $\frac{225}{100}$, which ultimately simplifies to the equivalent decimal value of 2.25 (**7.RP.A.1**). Students use complex fractions to represent non-whole number percents Module 3's focus on algebra prepares students to move from the visual models used for percents in Grade 6 to algebraic equations in Grade 7. They write equations to solve multi-step percent problems and relate their conceptual understanding to the representation: Quantity=Percent×Whole (**7.RP.A.2c**). Students solve percent increase and decrease problems with and without equations (**7.RP.A.3**). For instance, given a multi-step word problem where there is an increase of 20% and "the	**7.RP.A.1, 2c, 3** **A. Analyze proportional relationships and use them to solve real-world and mathematical problems.** **1.** Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction* $^{1/2}/_{1/4}$ *miles per hour, equivalently 2 miles per hour*. **2**. Recognize and represent proportional relationships between quantities. **c)** Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.* **3.** Use proportional relationships to solve multistep ratio and percent problems. Examples:	**MP.1 Make sense of problems and persevere in solving them.** Students make sense of percent problems by modeling the proportional relationship using an equation, a table, a graph, a double number line diagram, mental math, and factors of 100. When solving a multi-step percent word problem, students use estimation and number sense to determine if their steps and logic lead to a reasonable answer. Students know they can always find 1% of a quantity by dividing it by 100 or multiplying it by $\frac{1}{100}$, and they also know that finding 1% first allows them to then find other percents easily. For instance, if students are trying to find the	• Students understand that P percent is the number $\frac{P}{100}$ and that the symbol % means percent. • Students convert between a fraction, decimal, and percent, including percents that are less than 1% or greater than 100%. • Students write a non-whole number percent as a complex fraction. • Students understand that the whole is 100% and use the formula Part=Percent×Whole to problem-solve when given two terms out of three from the part, whole, and percent. • Students solve word problems involving percent using expressions, equations, and numeric and visual models. • Students use the context of a word problem to determine which of two	**ENY 7 Mod 4** **Topic A** Lessons 1-6

whole" equals $200, students recognize that $200 can be multiplied by 120%, or 1.2, to get an answer of $240. They use visual models such as a double number line diagram to justify their answers. In this case, 100% aligns to $200 in the diagram, and intervals of fifths are used (since 20%=15) to partition both number line segments to create a scale indicating that 120% aligns to $240. Topic A concludes with students representing 1% of a quantity using a ratio and then using that ratio to find the amounts of other percents. While representing 1% of a quantity and using it to find the amount of other percents is a strategy that will always work when solving a problem, students recognize that when the percent is a factor of 100, they can use mental math and proportional reasoning to find the amount of other percents in a more efficient way.	simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.	amount of money after 4 years in a savings account with an annual interest rate of 12% on an account balance of $300, they use the fact that 1% of 300 equals $\frac{300}{100}$, or $3; thus, 12% of 300 equals 12 of $3, or $1.50. $1.50 multiplied by 4 is $6 interest, and adding $6 to $300 makes the total balance, including interest, equal to $306. **MP.2 Reason abstractly and quantitatively.** Students use proportional reasoning to recognize that when they find a certain percent of a given quantity, the answer must be greater than the given quantity if they found more than 100% of it and less than the given quantity if they found less than 100% of it. Double number line models are used to visually represent proportional reasoning related to percents in problems such as the following: If a father has 70% more money in his savings account than his 25-year-old daughter has in her savings account,	quantities represents the whole. • Students find 100% of a quantity (the whole) when given a quantity that is a percent of the whole by using a variety of methods including finding 1%, equations, mental math using factors of 100, and double number line models. • Students solve word problems involving finding 100% of a given quantity with and without using equations. • Students solve percent problems when one quantity is a certain percent more or less than another. • Students solve percent problems involving a percent increase or decrease. • Students solve various types of percent problems by identifying the type of percent problem and applying appropriate strategies. • Students extend mental math practices to mentally calculate the part, the percent, or the whole in percent word problems.	

		and the daughter has $4,500, how much is in the father's account? Students represent this information with a visual model by equating 4,500 to 100% and the father's unknown savings amount to 170% of 4,500. Students represent the amount of money in the father's savings account by writing the expression 170100×4,500, or 1.7(4,500). When working with scale drawings, given an original two-dimensional picture and a scale factor as a percent, students generate a scale drawing so that each corresponding measurement increases or decreases by a certain percentage of measurements of the original figure. Students work backward to create a new scale factor and scale drawing when given a scale factor represented as a percent greater or less than 100%. For instance, given a scale drawing with a scale factor of 25%, students create a new scale		

		drawing with a scale factor of 10%. They relate working backward in their visual model to the following steps: (1) multiplying all lengths in the original scale drawing by 10.25 (or dividing by 25%) to get back to their original lengths, and then (2) multiplying each original length by 10% to get the new scale drawing. **MP.5 Use appropriate tools strategically.** Students solve word problems involving percents using a variety of tools, including equations and double number line models. They choose their model strategically. For instance, given that 75% of a class of learners is represented by 21 students, they recognize that since 75 is 34 of 100, and 75 and 21 are both divisible by 3, a double number line diagram can be used to establish intervals of 25's and 7's to show that 100% would correspond to 21+7, which equals 28. For percent problems that do not involve benchmark fractions,		

		decimals, or percents, students use math sense and estimation to assess the reasonableness of their answers and computational work. For instance, if a problem indicates that a bicycle is marked up 18% and it is sold at a retail price of $599, students are able to estimate by using rounded values such as 120% and $600 to determine that the solution that will represent the wholesale price of the bicycle must be in the realm of 600÷1.2, or 6,000÷12, to arrive at an estimate of $500. **MP.6 Attend to precision.** Students pay close attention to the context of the situation when working with percent problems involving a percent markup, markdown, increase, or decrease. They construct models based on the language of a word problem. For instance, a markdown of 15% on an $88 item is represented by 0.85(88); however, a markup of 15% is represented by 1.15(88). Students		

		attend to precision when writing the answer to a percent problem. If they are finding a percent, they use the % symbol in the answer or write the answer as a fraction with 100 as the denominator (or in an equivalent form). Double number line diagrams display correct segment lengths, and if a line in the diagram represents percents, it is either labeled as such or the percent sign is shown after each number. When stating the area of a scale drawing or actual drawing, students include the square units along with the numerical part of the answer.		
Topic B: Percent Problems Including More Than One Whole In Topic B, students understand and interpret the elements of increasingly complex real-world problems and directly connect elements in these contexts to concepts covered in Topic A (**7.RP.A.2**, **7.RP.A.3**, **7.EE.B.3**) as well as how the part, whole, and percent equation can be applied as such. The topic begins in Lesson 7, with students solving markup and markdown problems. They understand that the markup price will be more than the whole or more than 100% of the	**7.RP.A.1, 2c, 3** **A. Analyze proportional relationships and use them to solve real-world and mathematical problems.** **1.** Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction* $^{1/2}/_{1/4}$ *miles per hour, equivalently 2 miles per hour*.		• Students understand the terms original price, selling price, markup, markdown, markup rate, and markdown rate. • Students identify the original price as the whole and use their knowledge of percent and proportional relationships to solve multi-step markup and markdown problems. • Students understand equations for markup and	**Topic B** Lessons 7-11 Skip 8-9 too deep Lesson 10-11 (skip problems with percent error questions) After lesson 10 (teacher page 152) Included- in the student work book the

original price. And similarly, they know that the markdown price or discount price will be less than 100% of the whole. This conceptual understanding supports students' algebraic representations. To find a markup price, they multiply the whole by (1+m), where m is the markup rate, and to find a markdown price, they multiply the whole by (1−m), where m is the markdown rate. They write and solve algebraic equations, working backward, for instance, to find a price before a markup when given the percent increase and markup price. Students relate percent markup or markdown to proportional relationships as they consider cases where items of varying initial prices undergo a markup (or markdown). They create an equation, a table, and a graph relating the initial prices to the prices after markup (or markdown). They relate the constant of proportionality to the markup or markdown rate, m, using the value of (1+m) in the case of a markup or (1−m) in the case of a markdown. Students also identify and describe in context the meaning of the point (1,(1+m)) or (1,(1−m)) on the graph. Students continue to apply their conceptual understanding of the relationship between *part*, *whole*, and *percent* as they are introduced to *percent error* in Lesson 8. Additionally, they draw upon prior experiences with absolute value to make sense of the percent error formula and relate it to the elements of a word problem. Given an exact value, x, of a quantity and an approximate value, aa, of the quantity, students use absolute value to	**2**. Recognize and represent proportional relationships between quantities. **3.** Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error. **7.EE.B.3** **B. Solve real-life and mathematical problems using numerical and algebraic expressions and equations.** **3.** Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a*		markdown problems and use them to solve for unknown quantities in such scenarios. • Students solve simple interest problems using the formula I=Prt, where I= interest,= principal, r= interest rate, and t= time. • When using the formula I=Prt, students recognize that units for both interest rate and time must be compatible; students convert the units when necessary. • Students solve real-world percent problems involving tax, gratuities, commissions, and fees. • Students solve word problems involving percent using equations, tables, and graphs. • Students identify the constant of proportionality (tax rate, commission rate, etc.) in graphs, equations, and tables, and in the context of the situation.	lesson sprint (Fractional percent sprit) After Topic B Mid Module Assessment Mid Module Assessment

represent the *absolute error* as $\lvert a-x \rvert$, and then use that to compute the *percent error* with the formula: $\lvert a-x \rvert \div \lvert x \rvert \cdot 100\%$. Students understand that even when an exact value is not known, an estimate of the percent error can still be computed when given an inclusive range of values in which the exact value lies. In Lesson 10, students use the formula interest=principal×rate ×time to solve problems involving simple interest, and they relate principal to the whole, the interest rate to the percent, and the amount of interest to the part. When solving an interest problem, students pay close attention to the unit provided for the interest rate as well as the unit of time and are able to convert when necessary so that they remain compatible. Topic B concludes with Lesson 11, which involves percents related to other rates, such as tax, commission, and fees. Students apply their conceptual understanding of the part, whole, and percent to a real-life scenario related to the formation of a new sports team in a school district. In Lessons 10 and 11, students interpret and represent these proportional relationships through equations, graphs, and tables (**7.RP.A.1**, **7.RP.A.2**), recognizing where the constant of proportionality is present in their equations and graphs and connecting it to the value $(1+m)$ or $(1-m)$, where m is the rate given as a percentage.	*check on the exact computation*.			

Topic C: Scale Drawings In Lesson 12, students extend their understanding of scale factor from Module 1 to include scale factors represented as percents. Students know the scale factor to be the constant of proportionality, and they create scale drawings when given horizontal and vertical scale factors in the form of percents (**7.G.A.1**, **7.RP.A.2b**). In Lesson 13, students recognize that if Drawing B is a scale drawing of Drawing A, then one could also view Drawing A as being a scale drawing of Drawing B; they compute the scale factor from Drawing B to Drawing A and express it as a percentage. Also in this lesson, students are presented with three similar drawings—an original drawing, a reduction, and an enlargement—and, given the scale factor for the reduction (as a percentage of the original) and the scale factor for the enlargement (as a percentage of the original), students compute the scale factor between the reduced image and the enlarged image, and vice- versa, expressing each scale factor as a percentage. In Lesson 14, students compute the actual dimensions when given a scale drawing and the scale factor as a percent. To solve area problems related to scale drawings, in Lesson 15, students use the fact that an area, A', of a scale drawing is k^2 times the corresponding area, A, in the original picture (where k is the scale factor). For instance, given a scale factor of 25%, students convert to its fractional representation of $\frac{1}{4}$ and know that the area of the scale drawing will	**7.RP.A.2b** **A. Analyze proportional relationships and use them to solve real-world and mathematical problems.** **2.** Recognize and represent proportional relationships between quantities. **b)** Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. **7.G.A.1** **A. Draw construct, and describe geometrical figures and describe the relationships between them.** **1.** Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.	.	• Given a scale factor as a percent, students make a scale drawing of a picture or geometric figure using that scale, recognizing that the enlarged or reduced distances in a scale drawing are proportional to the corresponding distances in the original picture. • Students understand scale factor to be the constant of proportionality. • Students make scale drawings in which the horizontal and vertical scales are different. • Given Drawing 1 and Drawing 2 (a scale model of Drawing 1 with scale factor), students understand that Drawing 1 is also a scale model of Drawing 2 and compute the scale factor. • Given three drawings that are scale drawings of each other and two scale factors, students compute the other related scale factor. • Given a scale drawing, students compute the lengths in the actual picture using the scale factor. • Students solve area problems related to scale drawings and percent by using the fact that an	**Topic C** Lessons 12-15 (at teacher discretion) **Omit topic D but pull lesson for enrichment.** *7Mod 4 Lesson 18 Could use for an intro to probability if time allows. End of Module Assessment End of Module Assessment

be$(\frac{1}{4})^2$ or $\frac{1}{16}$ the area of the original picture and use that fact to problem solve.			area, A', of a scale drawing is k^2 times the corresponding area, A, in the original drawing, where k is the scale factor.	

Unit 5 – Statistics and Probability (ENY G7 Mod 5) Time line 15 days				
Students begin their study of probability, learning how to interpret probabilities and how to compute probabilities in simple settings. They also learn how to estimate probabilities empirically. Probability provides a foundation for the inferential reasoning developed in the second half of this module. Additionally, students build on their knowledge of data distributions that they studied in Grade 6, compare data distributions of two or more populations, and are introduced to the idea of drawing informal inferences based on data from random samples.				
Big Ideas	**Common Core State Standards**	**Standards for Mathematical Practice**	**Objectives**	**Materials/Resources Examples**
Topic A: Calculating and Interpreting Probabilities In Topic A, students begin a study of basic probability concepts (**7.SP.C.5**). They are introduced to the idea of a chance experiment and how probability is a measure of how likely it is that an event will occur. Working with spinners and other chance experiments, students estimate probabilities of outcomes (**7.SP.C.6**). In Lesson 1, students collect data they will use to estimate a probability in Lesson 2. Lesson 2 also provides additional opportunities to use data to estimate a probability. In Lesson 3, students are introduced to the terminology of probability, including *event*, *outcome*, and *sample space*. They are asked to think about chance experiments in terms of whether or not outcomes in the sample space are equally likely. In Lesson 4, they determine the sample space for a chance experiment and calculate the probabilities of events based on the sample space (**7.SP.C.7**). In this lesson, students also learn to assign probabilities to outcomes in a sample space when the outcomes are equally likely. They then calculate the probability of compound events that consist of more than a single outcome. This lesson leads	**7.SP.C.5, 6, 7, 8a, 8b** C. Investigate chance processes and develop, use, and evaluate probability models. **5.** Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event. **6.** Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times*. **7.** Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is	MP.2 Reason abstractly and quantitatively. Students reason quantitatively by posing statistical questions about variables and the relationship between variables. Students reason abstractly about chance experiments by analyzing possible outcomes and designing simulations to estimate probabilities. MP.3 Construct viable arguments and critique the reasoning of others. Students construct viable arguments by using sample data to explore conjectures about a population. Students critique the reasoning of other students as part of poster or similar presentations. MP.4 Model with mathematics. Students use probability models to describe outcomes of chance experiments.	• Students understand that a probability is a number between 0 and 1 that represents the likelihood that an event will occur. • Students interpret a probability as the proportion of the time that an event occurs when a chance experiment is repeated many times. • Students estimate probabilities by collecting data on an outcome of a chance experiment. • Students use given data to estimate probabilities. • Students determine the possible outcomes for simple chance experiments. • Given a description of a simple chance experiment, students determine the sample space for the experiment. • Given a description of a chance experiment and an event, students determine for which outcomes in the sample space the event will occur.	**ENY 7 Mod 5** **Topic A** Lessons 1-7

students to see that when outcomes are equally likely, the probability of an event is the number of outcomes in the event divided by the number of outcomes in the sample space. In Lesson 5, students begin to analyze chance experiments that have outcomes that are not equally likely. They calculate probabilities of various events by adding appropriate probabilities. Students learn in Lesson 6 to represent a sample space by a tree diagram and use the tree to calculate probabilities of compound events (**7.SP.C.8**). In Lesson 7, students calculate probabilities of compound events using sample spaces represented as lists of outcomes and presented as tree diagrams. This topic moves students from calculating and interpreting probabilities in simple settings into the lessons of Topic B, where students estimate probabilities empirically based on a large number of observations and by simulation.	not good, explain possible sources of the discrepancy. a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected. **b)** Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies? **8.** Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. **a)** Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. **b)** Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample	They evaluate probability models by calculating the theoretical probabilities of chance events, and by comparing these probabilities to observed relative frequencies. **MP.5** Use appropriate tools strategically. Students use simulation to approximate probabilities. Students use appropriate technology to calculate measures of center and variability. Students use graphical displays to visually represent distributions. **MP.6** Attend to precision. Students interpret and communicate conclusions in context based on graphical and numerical data summaries. Students make appropriate use of statistical terminology.	• Students distinguish between chance experiments with equally likely outcomes and chance experiments for which the outcomes are not equally likely. • Students calculate probabilities of events for chance experiments that have equally likely outcomes. • Students calculate probabilities for chance experiments that do not have equally likely outcomes. • Given a description of a chance experiment that can be thought of as being performed in two or more stages, students use tree diagrams to organize and represent the outcomes in the sample space. • Students calculate probabilities of compound events. • Students calculate probabilities of compound events.	

	space which compose the event.			
Topic B: Estimating Probabilities In Topic B, students estimate probabilities empirically and by using simulation. In Lesson 8, students make the distinction between a theoretical probability and an estimated probability. For a simple chance experiment, students carry out the experiment many times and use observed frequencies to estimate known theoretical probabilities. Students also consider chance experiments for which they cannot compute theoretical probabilities. In Lesson 9, students continue to collect data from a chance experiment and use it to estimate probabilities. Students compare these probabilities to theoretical probabilities from a model and then assess the plausibility of the model	**7.SP.C.6, 7a, 7b, 7SP.C.8** **C. Investigate chance processes and develop, use, and evaluate probability models**. **6.** Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times. **7.** Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. **a)** Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected. **b)** Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. For example, find the	**MP.2 Reason abstractly and quantitatively.** Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of	• Given theoretical probabilities based on a chance experiment, students describe what they expect to see when they observe many outcomes of the experiment. • Students distinguish between theoretical probabilities and estimated probabilities. • Students understand that probabilities can be estimated based on observing outcomes of a chance experiment. • Students compare estimated probabilities to those predicted by a probability model.	**Topic B** Lessons 8-9 After Topic B Mid Module Assessment Mid Module Assessment

	approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies? **8**. Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. c. Design and use a simulation to generate frequencies for compound events. For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?	quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects. **MP.4 Reason abstractly and quantitatively.** Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to		

		the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.		
Topic C: Random Sampling and Estimating Population Characteristics In Topics C and D, students focus on using random sampling to draw informal inferences about a population (**7.SP.A.1**, **7.SP.A.2**). In Topic C, they investigate sampling from a population (**7.SP.A.2**). They learn to estimate a population mean using numerical data from a random sample (**7.SP.A.2**). They also learn how to estimate a population proportion using categorical data from a random sample. In Topic D, students learn to compare two populations with similar variability. They learn to consider sampling variability when deciding if there is evidence that the means or the proportions of two populations are actually different (**7.SP.B.3**, **7.SP.B.4**). Topic C begins developing the concept of generalizing from a sample to a larger population. In **Lesson 13**, students are introduced to the following terminology: population, sample, population characteristic, and sample statistic. Students distinguish between the population and a sample and between a population characteristic and a sample statistic as they investigate statistical questions.	**7.SP.A.1 & 2** **A. Use random sampling to draw inferences about a population.** **1.** Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences. **2.** Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.	**MP.3:** Be sure students answer the questions as if they had no other source available; they could not go to the Internet and ask for the average home cost, for example. They would have to figure out how to get enough information to estimate an average cost. Pose questions from this exercise one at a time, and allow for multiple responses. As you discuss the answers, point out the difference between a population and a sample and how that might be related to each part of this exercise. A population is the entire set of objects (people, animals, plants, etc.) from which data might be collected. A sample is a subset of the population. Consider organizing a table similar to the following for selected parts of this exercise as students discuss their answers.	• Students differentiate between a population and a sample. • Students differentiate between a population characteristic and a sample statistic. • Students investigate statistical questions that involve generalizing from a sample to a larger population. • Students understand that how a sample is selected is important if the goal is to generalize from the sample to a larger population. • Students understand that random selection from a population tends to produce samples that are representative of the population. • Students use data from a random sample to estimate a population mean. • Students understand the term "sampling variability" in the context of estimating a population mean.	**Topic C** Lessons 13-14 Lessons 17 Lessons 19-20 0mit lesson 15,16, 18 (use for enrichment)

In **Lesson 14,** students learn the importance of random sampling and of using a random mechanism in the sample selection process. Students see a more formal introduction to "sampling variability" in **Lesson 17,** where several samples are randomly selected from the sample population. They compute sample means and use collected data to develop a sense of variation in the values of a sample mean and an understanding of how this variability is related to the size of a sample**. In Lesson 19**, they develop an understanding of the term sampling variability in the context of estimating a population mean. **In Lesson 20**, they estimate a population proportion using categorical data from a random sample.		**MP.2:** Here, the sample means for the class are combined in order to form a visual illustration of sampling variability (in the form of a dot plot of the class's sample means). Students are asked to show how the pattern in the dot plot supports the notion of sampling variability. Students will need to refer to the dot plot in their work in the next lesson, so every student will need to complete the dot plot. **MP.6:** The standards for this lesson expect students will be involved in obtaining their own sample and using the proportion derived from their sample to estimate the population proportion. By examining the distribution of sample proportions from many random samples, students see that sample proportions tend to cluster around the value of the population proportion. Students attend to precision by carefully describing how they use samples to describe the population.	• Students understand the term *sampling variability* in the context of estimating a population proportion. • Students know that increasing the sample size decreases sampling variability. • Students use data from a random sample to estimate a population proportion.	

Topic D: Comparing Populations Students learn to compare two populations with similar variability. They learn to consider sampling variability when deciding if there is evidence that the means or the proportions of two populations are actually different (**7.SP.B.3**,**7.SP.B.4**). In Topic D, students learn to compare two populations with similar variability. They learn to consider sampling variability when deciding if there is evidence that population means or population proportions are actually different. **In Lesson 21**, students work with random samples from two different populations that have similar variability. They decide if there is evidence that the means of the two populations are actually different**. In Lesson 22**, students describe the difference in sample means from populations with similar variability by using a multiple of the measure of variability. They explore how big the difference in sample means would need to be in order to indicate a difference in population means. This lesson sets the stage for drawing informal conclusions about the difference between two populations' means from populations with similar variability**.** **Lesson 23**, again, uses random samples to draw informal inferences about the differences in means of two populations. Students work with examples in which there is a meaningful difference in population means and also with examples in which there is no evidence of a	**7.SP.B.3 & 4** **B. Draw informal comparative inferences about two populations.** **3.** Informally assess the degree of visual overlap of two numerical data distributions with similar variability, measuring the difference between the centers by expressing it as a multiple of a measure of variability. For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable. **4.** Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.	**MP.3:** Construct viable arguments and critique the reasoning of others. Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and — if there is a flaw in an Argument — explain what it is. Later, students learn to	• Students use data from random samples to draw informal inferences about the difference in population means. • Students express the difference in sample means as a multiple of a measure of variability. • Students understand that a difference in sample means provides evidence that the population means are different if the difference is larger than what would be expected as a result of sampling variability alone • Students understand that a meaningful difference between two sample means is one that is greater than would have been expected due to just sampling variability.	**Topic D** Lessons 21-23 After Topic D End of Module Assessment End of Module Assessment

meaningful difference in population means.		determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.		

Unit 6 – Geometry (ENY Grade 7 Module 6, ENY Grade 8 Modules 2 & 3) **Time line – 50 days**
This unit is a compilation of Grade 7 Geometry (Engage Module 6 Topic A) and Grade 8 Geometry (Engage Module 2 and 3). Students delve further into several geometry topics they have been developing over the years. Grade 7 presents some of these topics, (e.g., angles, area, surface area, and volume) in the most challenging form students have experienced yet. Module 6 assumes students understand the basics. The goal is to build a fluency in these difficult problems. Students learn about translations, reflections, and rotations in the plane and, more importantly, how to use them to precisely define the concept of congruence. Throughout Topic A, on the definitions and properties of the basic rigid motions, students verify experimentally their basic properties and, when feasible, deepen their understanding of these properties using reasoning. All the lessons of Topic B demonstrate to students the ability to sequence various combinations of rigid motions while maintaining the basic properties of individual rigid motions. Students learn that congruence is just a sequence of basic rigid motions in Topic C, and Topic D begins the learning of Pythagorean Theorem. Students learn about dilation and similarity and apply that knowledge to a proof of the Pythagorean Theorem based on the Angle-Angle criterion for similar triangles. The module begins with the definition of dilation, properties of dilations, and compositions of dilations. One overarching goal of this module is to replace the common idea of "same shape, different sizes" with a definition of similarity that can be applied to geometric shapes that are not polygons, such as ellipses and circles.

Big Ideas	Common Core State Standards	Standards for Mathematical Practice	Objectives	Materials/Resources Examples
G7 M6 **Topic A: Unknown Angles** In Topic A, students solve for unknown angles. The supporting work for unknown angles began in Grade 4, Module 4 (**4.MD.C.5–7**), where all of the key terms in this Topic were first defined, including: adjacent, vertical, complementary, and supplementary angles, angles on a line, and angles at a point. In Grade 4, students used those definitions as a basis to solve for unknown angles by using a combination of reasoning (through simple number sentences and equations), and measurement (using a protractor). For example, students learned to solve for a missing angle in a pair of supplementary angles where one angle measurement is known. In Grade 7, Module 3, students studied how expressions and equations are an efficient way to solve problems. Two lessons were dedicated to applying the	**7.G.B.5** **B.** Solve real-life and mathematical problems involving angle measure, area, surface area, and volume. **5.** Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.	MP.1 **Make sense of problems and persevere in solving them.** This mathematical practice is particularly applicable for this module, as students tackle multi-step problems that require them to tie together knowledge about their current and former topics of study (i.e., a real-life composite area question that also requires proportions and unit conversion). In many cases, students will have to make sense of new and	• Students solve for unknown angles in word problems and in diagrams involving complementary and supplementary angles • Students solve for unknown angles in word problems and in diagrams involving complementary, supplementary, vertical, and adjacent angles • Students solve for unknown angles in word problems and in diagrams involving all learned angle facts. • Students solve for unknown angles in word problems and in diagrams involving all learned angle facts.	**ENY 7 Mod 6** **Topic A** Lessons 1-4

properties of equality to isolate the variable in the context of missing angle problems. The diagrams in those lessons were drawn to scale to help students more easily make the connection between the variable and what it actually represents. Now in Module 6, the most challenging examples of unknown angle problems (both diagram-based and verbal) require students to use a synthesis of angle relationships and algebra. The problems are multi-step, requiring students to identify several layers of angle relationships and to fit them with an appropriate equation to solve. Unknown angle problems show students how to look for, and make use of, structure (MP.7). In this case, they use angle relationships to find the measurement of an angle.		different contexts and engage in significant struggle to solve problems. **MP.7 Look for and make use of structure**. Students must examine combinations of angle facts within a given diagram in Topic A to create an equation that correctly models the angle relationships. If the unknown angle problem is a verbal problem, such as an example that asks for the measurements of three angles on a line where the values of the measurements are consecutive numbers, students will have to create an equation without a visual aid and rely on the inherent structure of the angle fact.		
G8 M2 Topic C: Congruence and Angle Relationships In Topic C, which introduces the definition and properties of congruence, students learn that congruence is just a sequence of basic rigid motions. The fundamental properties shared by all the basic rigid motions are then inherited by congruence: Congruence moves lines	**8.G.A.2 & 5** **A. Understand congruence and similarity using physical models, transparencies, or geometry software.** **3.** Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations;	**MP.2 Reason abstractly and quantitatively.** This module is rich with notation that requires students to decontextualize and contextualize throughout. Students work with figures and their transformed	• Students know the definition of congruence and related notation. Students know that to prove two figures are congruent there must be a sequence of rigid motions that maps one figure onto the other. • Students know the basic properties of congruence	**Grade 8 Module 2** **Topic C** Lessons 11-14

to lines and angles to angles, and it is both distance- and angle-preserving (Lesson 11). In Grade 7, students used facts about supplementary, complementary, vertical, and adjacent angles to find the measures of unknown angles (**7.G.B.5**). This module extends that knowledge to angle relationships that are formed when two parallel lines are cut by a transversal. In Topic C, on angle relationships related to parallel lines, students learn that pairs of angles are congruent because they are angles that have been translated along a transversal, rotated around a point, or reflected across a line. *Students use this knowledge of angle relationships in Lessons 13 and 14 to show why a triangle has a sum of interior angles equal to 180° and why the measure of each exterior angle of a triangle is the sum of the measures of the two remote interior angles of the triangle.*	given two congruent figures, describe a sequence that exhibits the congruence between them. **5.** Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.	images using symbolic representations and need to attend to the meaning of the symbolic notation to contextualize problems. Students use facts learned about rigid motions in order to make sense of problems involving congruence. **MP.3 Construct viable arguments and critique the reasoning of others.** Throughout this module, students construct arguments around the properties of rigid motions. Students make assumptions about parallel and perpendicular lines and use properties of rigid motions to directly or indirectly prove their assumptions. Students use definitions to describe a sequence of rigid motions to prove or disprove congruence. Students build a logical progression of statements to show relationships between angles of parallel lines cut by a transversal, the angle sum of triangles, and properties of polygons like rectangles and parallelograms.	are similar to the properties for all three rigid motions (translations, rotations, and reflections). • Students know that corresponding angles, alternate interior angles, and alternate exterior angles of parallel lines are equal. Students know that when these pairs of angles are equal, then lines are parallel. • Students know that corresponding angles of parallel lines are equal because of properties related to translation. Students know that alternate interior angles of parallel lines are equal because of properties related to rotation. • Students present informal arguments to draw conclusions about angles formed when parallel lines are cut by a transversal. • Students know the Angle Sum Theorem for triangles; the sum of the interior angles of a triangle is always 180 degrees. • Students present informal arguments to draw conclusions about the angle sum of a triangle. • Students know a third informal proof of the angle sum theorem.	

		MP.5 Use appropriate tools strategically. This module relies on students' fundamental understanding of rigid motions. As a means to this end, students use a variety of tools but none as important as an overhead transparency. Students verify experimentally the properties of rigid motions using physical models and transparencies. Students use transparencies when learning about translation, rotation, reflection, and congruence in general. Students determine when they need to use the transparency as a tool to justify conjectures or when critiquing the reasoning of others. **MP.6 Attend to precision**. This module begins with precise definitions related to transformations and statements about transformations being distance- and angle-preserving. Students are expected to attend to the precision of these definitions and statements consistently and appropriately as	• Students know how to find missing interior and exterior angle measures of triangles and present informal arguments to prove their answer is correct	

		they communicate with others. Students describe sequences of motions precisely and carefully label diagrams so that there is clarity about figures and their transformed images. Students attend to precision in their verbal and written descriptions of rays, segments, points, angles, and transformations in general.		
G8 M2 Topic A: **Definitions and Properties of the Basic Rigid Motions** Throughout Topic A, on the definitions and properties of the basic rigid motions, students verify experimentally their basic properties and, when feasible, deepen their understanding of these properties using reasoning. In particular, what students learned in Grade 4 about angles and angle measurement (**4.MD.C.5**) will be put to good use here. They learn that the basic rigid motions preserve angle measurements, as well as segment lengths.	**8.G.A.1** **A. Understand congruence and similarity using physical models, transparencies, or geometry software.** **1.** Verify experimentally the properties of rotations, reflections, and translations: a. Lines are taken to lines, and line segments to line segments of the same length. b. Angles are taken to angles of the same measure. c. Parallel lines are taken to parallel lines.		• Students are introduced to vocabulary and notation related to rigid motions (e.g., transformation, image, and map). • Students are introduced to transformations of the plane and learn that a rigid motion is a transformation that is distance preserving. • Students use transparencies to imitate a rigid motion that moves or maps one figure to another figure in the plane. • Students perform translations of figures along a specific vector. Students label the image of the figure using appropriate notation. • Students learn that a translation maps lines to lines, rays to rays, segments to segments,	**Grade 8 Module 2** **Topic A** Lessons 1 - 6

			and angles to angles. Students learn that translations preserve lengths of segments and degrees of angles. • Students learn that when lines are translated they are either parallel to the given line, or the lines coincide. • Students learn that translations map parallel lines to parallel lines. • Students know the definition of reflection and perform reflections across a line using a transparency. • Students show that reflections share some of the same fundamental properties with translations (e.g., lines map to lines, angle and distance preserving motion, etc.). Students know that reflections map parallel lines to parallel lines. • Students know that for the reflection across a line *L*, then every point *P*, not on *L*, *L* is the bisector of the segment joining *P* to its reflected image *P'*. • Students know how to rotate a figure a given degree around a given center. • Students know that rotations move lines to lines, rays to rays,	

			segments to segments, and angles to angles. Students know that rotations preserve lengths of segments and degrees of measures angles. Students know that rotations move parallel lines to parallel lines. • Students learn that a rotation of 180 degrees moves a point on the coordinate plane (*a*, *b*), to (-*a*, -*b*). • Students learn that a rotation of 180 degrees around a point, not on the line, produces a line parallel to the given line.	
G8 M2 Topic B: Sequencing the Basic Rigid Motions Topic B is a critical foundation to the understanding of congruence. All the lessons of Topic B demonstrate to students the ability to sequence various combinations of rigid motions while maintaining the basic properties of individual rigid motions. Lesson 7 begins this work with a sequence of translations. Students verify experimentally that a sequence of translations have the same properties as a single translation. Lessons 8 and 9 demonstrate sequences of reflections and translations and sequences of rotations. The concept of sequencing a combination of all three rigid motions is introduced in Lesson 10; this paves the way for the study of congruence in the next topic.	**8.G.A.2 & 5** **A. Understand congruence and similarity using physical models, transparencies, or geometry software.** **2.** Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them. **5.** Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the		• Students learn about the sequence of transformations (one move on the plane followed by another) and that a sequence of translations enjoy the same properties as a single translation with respect to lengths of segments and degrees of angles. • Students learn that a translation along a vector followed by another translation along the same vector in the opposite direction can move all points of a plane back to its original position.	**Grade 8 Module 2** **Topic B** Lessons 7 - 10 **Mid-Module Assessment**

	angle-angle criterion for similarity of triangles. For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.		• Students learn that the reflection is its own inverse transformation. • Students understand that a sequence of a reflection followed by a translation is not equal to a translation followed by a reflection. • Students learn that the reflection is its own inverse transformation. • Students understand that a sequence of a reflection followed by a translation is not equal to a translation followed by a reflection. • Students describe a sequence of rigid motions to map one figure onto another.	
G8 M2 Topic D: The Pythagorean Theorem Topic D introduces the Pythagorean theorem. Students are shown the “square within a square” proof of the Pythagorean theorem. The proof uses concepts learned in previous topics of the module, i.e., the concept of congruence and concepts related to degrees of angles. Students begin the work of finding the length of a leg or hypotenuse of a right triangle using $a^2 + b^2 = c^2$.	**8.G.B.6 & 7** **B. Understand and apply the Pythagorean Theorem.** **6.** Explain a proof of the Pythagorean Theorem and its converse. **7.** Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.		• Students will know the Pythagorean Theorem and be shown an informal proof of the theorem. • Students will use the Pythagorean Theorem to find the length of the hypotenuse of a right triangle. • Students use the Pythagorean Theorem to determine missing side lengths of right triangles.	**Grade 8 Module 2** **Topic D** Lessons 15 and 16
G8 M3 Topic A: Dilation Students describe the effect of dilations on two-dimensional figures in	**8.G.A.3** **A.** Understand congruence and similarity using physical models,	**MP.3 Construct viable arguments and critique the reasoning**	• Students learn the definition of dilation and why “same shape” is not	**Grade 8 Module 3** **Topic A** Lessons 1 - 7

general and using coordinates. Building on prior knowledge of scale drawings (**7.G.A.1**), Module 3 demonstrates the effect dilation has on a figure when the scale factor is greater than zero but less than one (shrinking of figure), equal to one (congruence) and greater than one (magnification of figure). Once students understand how dilation transforms figures in the plane, they examine the effect that dilation has on points and figures in the coordinate plane. Beginning with points, students learn the multiplicative effect that dilation has on the coordinates of an ordered pair. Then students apply the knowledge about points to describe the effect dilation has on figures in the coordinate plane, in terms of their coordinates.	transparencies, or geometry software. **3.** Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.	**of others**. Many times in this module, students are exposed to the reasoned logic of proofs. Students are called on to make conjectures about the effect of dilations on angles, rays, lines, and segments, and then they must evaluate the validity of their claims based on evidence. Students also make conjectures about the effect of dilation on circles, ellipses, and other figures. Students are encouraged to participate in discussions and evaluate the claims of others. **MP.4 Model with mathematics**. This module provides an opportunity for students to apply their knowledge of dilation and similarity in real-world applications. Students will use shadow lengths and a known height to find the height of trees, the distance across a lake, and the height of a flagpole. **MP.6 Attend to precision**. To communicate precisely, students will use clear definitions in	good enough to say two figures are similar. • Students know that dilations magnify and shrink figures. • Students learn how to use a compass and a ruler to perform dilations. • Students learn that dilations map lines to lines, segments to segments, and rays to rays. Students know that dilations are degree preserving. • Students know that dilations map circles to circles and ellipses to ellipses with the same shape. • Students know that to shrink or magnify a dilated figure back to its original size from center *O* with scale factor *r* you must dilate the figure by a scale factor of 1/*r*. • Students experimentally verify the properties related to the Fundamental Theorem of Similarity (FTS). • Students verify the converse of the Fundamental Theorem of Similarity experimentally. • Students apply the Fundamental Theorem of Similarity to find the location of dilated points on the plane.	

		discussions with others and in their own reasoning with respect to similar figures. Students will use the basic properties of dilations to prove or disprove claims about a pair of figures. Students will incorporate their knowledge about basic rigid motions as it relates to similarity, specifically in the description of the sequence that is required to prove two figures are similar. MP.8 **Look for and express regularity in repeated reasoning**. Students will look at multiple examples of dilations with different scale factors. Then students explore dilations to determine what scale factor to apply to return a figure dilated by a scale factor r to its original size.	• Students describe the effect of dilations on two-dimensional figures using coordinates. • Students know an informal proof of why dilations are degree-preserving transformations. • Students know an informal proof of why dilations map segments to segments, lines to lines, and rays to rays.	
G8 M3 Topic B: Similar Figures Topic B begins with the definition of similarity and the properties of similarity. In Lesson 8, students learn that similarities map lines to lines, change the lengths of segments by factor r and are angle-preserving.	**8.G.A.4** **A. Understand congruence and similarity using physical models, transparencies, or geometry software.** **4.** Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and		• Students know the definition of similar and why dilation alone is not enough to determine similarity. • Given two similar figures, students describe the sequence of a dilation and a congruence that would map one figure onto the other.	**Grade 8 Module 3** **Topic B** Lesson 8 - 12

In Lesson 9, investigates additional properties about similarity; first, students learn that congruence implies similarity (e.g., congruent figures are also similar). Next, students learn that similarity is symmetric (e.g., if figure A is similar to figure B, then figure B is similar to figure A) and transitive (e.g., if figure A is similar to figure B, and figure B is similar to figure C, then figure A is similar to figure C). Finally, students learn about similarity with respect to triangles. Lesson 10 provides students with an informal proof of the angle-angle (AA) criterion for similarity of triangles. Lesson 10 also provides opportunities for students to use the AA criterion to determine if a pair of triangles is similar. In Lesson 11, students use what they know about similar triangles and dilation to find an unknown side length of one triangle. Since students know that similar triangles have side lengths that are equal in ratio (specifically equal to the scale factor), students verify whether or not a pair of triangles is similar by comparing their corresponding side lengths. In Lesson 12, students apply their knowledge of similar triangles and dilation to real-world situations. For example, students use the height of a person and the height of his shadow to determine the height of a tree. Students may also use their knowledge to determine the distance across a lake, the height of a building, and the height of a flagpole.	dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them. **5.** Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.		• Students know that similarity is both a symmetric and a transitive relation. • Students know an informal proof of the Angle-Angle (AA) criterion for similar triangles. • Students present informal arguments as to whether or not triangles are similar based on Angle-Angle criterion. • Students present informal arguments as to whether or not two triangles are similar. • Students practice finding lengths of corresponding sides of similar triangles. • Students use properties of similar triangles to solve real-world problems.	

G8 M3 Topic C: Pythagorean Theorem Students have another opportunity for students to learn about the Pythagorean Theorem and its applications in these extension lessons. With the concept of similarity firmly in place, students are shown a proof of the Pythagorean Theorem that uses similar triangles.	**8.G.B.6 & 7** **B. Understand and apply the Pythagorean Theorem.** **6.** Explain a proof of the Pythagorean Theorem and its converse. **7.** Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.		• Students practice applying the Pythagorean Theorem to find lengths of right triangles in two dimensions. • Students illuminate the converse of the Pythagorean Theorem through computation of examples and counterexamples. • Students apply the theorem and its converse to solve problems.	**Grade 8 Module 3** **Topic C** Lesson 13 and 14